

Helioseismic test of non-homologous solar radius changes with the 11-year activity cycle

S. Lefebvre

SAP/DAPNIA/DSM CEA Saclay, L'Orme des Merisiers Bât. 709, Gif sur Yvette cedex, 91191, France

UMR 7158 CEA-CNRS-Université Paris VII, 91191 Gif sur Yvette cedex, France

`sandrine.lefebvre@cea.fr`

A. G. Kosovichev

W. W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305-4085, USA

and

J. P. Rozelot

Observatoire de la Côte d'Azur, GEMINI dept., Avenue Copernic, Grasse, 06130, France

ABSTRACT

Recent models of variations of the Sun's structure with the 11-year activity cycle by Sofia et al. (2005) predict strong non-homologous changes of the radius of subsurface layers, due to subsurface magnetic fields and field-modulated turbulence. According to their best model the changes of the surface radius may be 1000 times larger than those at the depth of 5 Mm. We use f-mode oscillation frequency data from the MDI instrument of Solar and Heliospheric Observatory (SOHO) and measurements of the solar surface radius variations from SOHO and ground-based observatories during solar cycle 23 (1996-2005) to put constraints on the radius changes. The results show that the above model overestimates the change of the radius at the surface relative to the change at 5Mm.

Subject headings: Sun: helioseismology — Sun: oscillations — Sun: activity — Sun: interior

1. Introduction

For the last 30 years, the question of whether the solar radius changes with time, and with the 11-year activity cycle, in particular, has been the subject of many controversies. Using different techniques, several series of measurements have been made leading so far to inconsistent results. Even if astrolabe measurements can be considered as extracted from the same statistical population (Badache-Damiani & Rozelot 2006), Laclare et al. (1996) and Reis Neto et al. (2003) reported a variation of the solar radius in antiphase with the solar cycle while Noël (2004) showed the opposite. In addition, drift-time measurements made photoelectrically from CCD transits at Izaña and Locarno did not show cycle-dependent variations in excess of ± 50 mas¹ (Wittmann & Bianda 2000). Using SOHO/MDI images, Kuhn et al. (2004) found no significant variations at any level above 7 mas, which is significantly lower than any variation reported from ground-based instruments (astrolabes) ranging from 50 to 200 mas. Using the Solar Disk Sextant experiment aboard stratospheric balloons, Sofia et al. (1994) indicated an antiphase variation between the solar radius and the 11-year cycle.

Helioseismic methods can also be used to study the variations with the cycle: in this case, a seismic radius is defined from oscillation frequencies of solar f -modes (Schou et al. 1997; Antia 1998). The seismic radius is an interesting probe of the subsurface layers to a depth of about 15 Mm, as recently published by Dziembowski & Goode (2004) and Lefebvre & Kosovichev (2005) (hereafter LK05). Using properties of f -mode frequencies, LK05 showed that the variations of the solar seismic radius were nonuniform with depth. More precisely, LK05 reported a non-monotonic change in the position of the subsurface layers: the radius changes between approximately 0.97 and $0.99 R_{\odot}$ were in phase with the solar cycle while the radius of the shallower layers above $0.99 R_{\odot}$ up to the surface changed in antiphase. In general, the results are consistent with previous conclusions that solar-cycle variations in the solar radius are confined in the outermost layers of the Sun (Antia & Basu 2004; Dziembowski & Goode 2005).

Using an evolution code of the Sun, Sofia et al. (2005) (hereafter SBDLT05) also found a non-homologous variation of the solar radius with depth and a radius change at the surface in antiphase with the solar activity cycle. Using the Yale Rotating Evolution Code (YREC; (Winnick et al. 2002) and including the effects of a variable magnetic field and field-modulated turbulence, SBDLT05 found a monotonic variation of the solar radius, sharply increasing in amplitude by a factor of approximately 1000 from a depth of 5 Mm to the solar surface between 1996 and 2000.

¹the mas is the milliarcsecond and is equal to about 0.725 km on the Sun

Both, the helioseismology and model results show that the solar-cycle variations of the solar radius with depth in the subsurface layers are non-homologous, and that the radius change at the surface is in antiphase with the solar activity.

However, there are also discrepancies between the model and helioseismic inferences concerning the amplitude of these variations and the behavior of the changes in the subsurface stratification in the first 15 Mm. LK05 found an oscillation and a change of phase in the first 15 Mm with variations not exceeding 30 km, whereas SBDLT05 found a monotonic variation in the first 5 Mm with a factor approximately up to 1000 between the depth at 5 Mm and the surface. However, LK05 showed that the helioseismic inferences are not very sensitive to the surface changes. Therefore, to make a quantitative test of the model prediction in addition to the helioseismology data we use measurements of the solar surface radius from the ground and space.

2. Computations and updated results concerning the inversion of f -mode frequencies

As in LK05, data used in this study are frequencies of solar oscillation modes from 72 day MDI observing runs². Only f -modes for the period 1996-2005 have been selected. The oscillation modes, common for the whole period, are extracted to finally obtain 148 modes ranging from $l = 125$ to $l = 285$. We will here simply make a summary of the formalism used in LK05, and the reader is invited to see that paper for detailed explanations.

A relation between the relative frequency variations $\delta\nu/\nu$ for f -modes and the associated Lagrangian perturbation of the radius $\delta r/r$ of subsurface layers has been established by Dziembowski & Goode (2004):

$$\left(\frac{\delta\nu}{\nu}\right)_l = -\frac{3l}{2\omega^2 I} \int dI \frac{g}{r} \frac{\delta r}{r} \quad (1)$$

where l is the degree of the f -modes, I is the moment of inertia, ω is the eigenfrequency and g is the gravity acceleration. This equation is the starting point for our inversion that will permit us to obtain the variation of the position of the subsurface layers determined by δr and $\delta\nu/\nu$. For these calculations, we used model S (Christensen-Dalsgaard et al. 1996) calibrated to the seismic radius $R_\odot = 6.9599 \times 10^5$ km, and the standard Regularized Least-Square technique (Tikhonov & Arsenin 1977) (as we have an ill-posed inverse problem).

²These files have been computed by J. Schou and are available on [http://quake.stanford.edu/\\$\sim\\$jschou/anavw72z/](http://quake.stanford.edu/\simjschou/anavw72z/)

For the inversions, as in LK05, we select f-modes of angular degree ℓ below 250, for which near-surface magnetic and turbulence effects are not important (Lefebvre et al. 2006).

The inversion of Eq. 1 leads to the results presented in Figure 1 which is an update of Figure 3 of LK05. The figure presents the variation δr for each year: every difference has been computed relative to the reference year of 1996. This updated graph presents the same features than the original one published in LK05: 1) No significant changes in the variation of the subsurface layers' depth below $0.97 R_{\odot}$ with a maximal amplitude of about 10 km at $\simeq 0.985 R_{\odot}$; 2) Non-monotonic changes in the stratification, with the inner layer varying in phase with the solar activity cycle and the outer layer evolving almost in antiphase with the solar cycle with a maximal amplitude of about 26 ± 6 km at about $0.995 R_{\odot}$.

The new result is the radius changes are not precisely in antiphase with the 11-year cycle. The results for 2005 indicate there might be an additional phase lag between the radius change and activity. The 2005 data show the same behavior as 1997 data of the previous solar minimum but the new activity minimum is not reached yet. Significance of this result needs to be checked with future data; if confirmed it may have interesting implications.

3. Comparison of model predictions with the data

In this section, we compare our results with the model results obtained by SBDLT05. Figure 3 of SBDLT05 shows the radius change as a function of depth below the photosphere relative to the radius change at the depth of 5 Mm for their model 4, which includes both temporal variation of magnetic field and turbulence. However, the model does not provide an absolute value for the radius variations. Therefore, we have to calibrate the model to obtain the best match to the helioseismic f-mode measurements.

The principle is to use Eq. 1 in a direct way by assuming a value of $|\Delta R_{5Mm}|$, obtaining $\frac{\delta r}{r}$ from model 4, and then calculating $\frac{\delta \nu}{\nu}$ from Eq. 1. Then, we search for the $|\Delta R_{5Mm}|$ value, which provides the best match to the observed changes of f -mode frequencies between 1996 and 2000. For the reference we use model S (Christensen-Dalsgaard et al. 1996) calibrated with the seismic radius ($R_{\odot} = 6.9599 \times 10^5$ km), the same as in the YREC model. The first step was to digitize the curve at the top of Figure 3 of SBDLT05 and transform to the fractional radius. The panels of Figure 2 show: a) the digitized curve as given by SBDLT05; b) the ΔR computed with different $|\Delta R_{5Mm}|$ ranging from 0.3 to 1 km; c) the associated $\Delta R/R$; and finally, d) computed $\Delta \nu/\nu$ for the different $\Delta R/R$ (color curves) in comparison with the $\Delta \nu/\nu$ observed between 1996 and 2000 (points with error bars).

Only few sample curves are plotted in panel d) of Figure 2, and a study using the χ^2 parameter shows that the minimum of χ^2 is obtained for $\Delta R_{5Mm} = 0.65$ km, which gives a value for ΔR at the surface of approximately 600 km. However, such high variation of the solar radius has never been observed in direct solar limb observations. In particular, Kuhn et al. (2004) put an upper limit of 7 mas, or just 5 km. This means that the theoretical model of the solar-cycle variations cannot satisfy both observational constraints, from helioseismology and from the limb measurements. To fit the f-mode helioseismology data it must have the surface radius variations by a factor 100 greater than the observational upper limit from SOHO/MDI (Kuhn et al. 2004). In this case, it also exceeds the observational limit from ground-based measurements by a factor of 4–20. Vice versa, if the model radius is within the observational limit then the model cannot explain the f-mode data.

Thus, we conclude that, according to the mathematical formalism previously described, the strong non-homologous variation of the subsurface solar radius proposed by SBDLT05 is not compatible with the variations of the f -mode frequencies and the surface radius, observed between 1996 and 2000. These computations explain why our results published in LK05 and updated here are in contradiction in amplitude and behavior with the results of SBDLT05. Of course our computations are completely based on Eq. 1 established by Dziembowski & Goode (2004) and on the hypothesis that the temporal variation of f -mode frequencies are mainly due to changes in the subsurface stratification during the solar activity cycle. On the other hand, the model proposed by SBDLT05 is based on the YREC code (Winnick et al. 2002) into which the effects of magnetic fields and turbulence have been included. But, as they said, the location, magnitude, and temporal behavior of the internal field are not known which imply to make assumptions on their treatment. We suggest that the difference obtained with our results could come from their treatment of the turbulence and magnetic fields in the subsurface layers. Overcoming this discrepancy between the theory and observations will help to understanding of the complicated magnetic and turbulence effects below the visible surface of the Sun.

4. Discussion

In this paper, we have shown that the solar-cycle variations of the solar radius that increase by a factor of approximately 1000 from a depth of 5 Mm to the solar surface as published by Sofia et al. (2005) (SBDLT05) are not consistent with the f -mode frequencies variations between 1996 and 2000 and the upper limit on the solar radius change from the simultaneous limb measurements. Using their model and the relationship between f -mode frequencies and solar radius variations, we obtained a variation at the solar surface

of about 600 km which is observed neither with ground-based nor space instruments. The model of Sofia et al. (2005) includes the effects of a variable dynamo magnetic field and of a field-modulated turbulence with general assumptions on their magnitude and temporal behavior (based on an observed luminosity change of 0.1% and the shape of the activity cycle) because of the unknown location, magnitude and temporal behavior of these internal fields. Nevertheless, we are still cautious on our above results because (i) the near-surface effects of turbulence and magnetic fields are not treated in our approach and (ii) the lack of very-high f -mode degrees are necessary to understand the very outer layers (approximately above 3 Mm). So we do not exclude a bigger variation of the position of these layers than those plotted in Figure 1 and that the solar radius at the photosphere by limb observations (as measured by present instruments and future astrometric satellites) could be larger than the seismic radius issued from f -mode frequencies studies. Hence, we agree with SBDLT05 and the necessity to measure precisely the variations of the solar radius from space.

This work utilizes data from SOHO/MDI and we thank J. Schou for providing the frequencies files. SOHO is a project of international cooperation between ESA and NASA. MDI is supported by NASA grant NAG 5-10483 to Stanford University. S. Lefebvre is supported by a CNES post-doctoral grant in SAp. She would like to thank Sylvaine Turck-Chièze for useful discussions on this field. The authors wish also to cordially thank both referees for very useful suggestions on the manuscript.

REFERENCES

- Antia, H. M. 1998, *A&A*, 330, 336
- Antia, H. M., & Basu, S. 2004, in *ESA-SP 559, Proc. SOHO 14/GONG 2004 Workshop: Helio- and Asteroseismology: Towards a Golden Future*, ed. D. Danesy, 301
- Badache-Damiani, C., & Rozelot, J. P. 2006, *MNRAS*, 369, 83
- Christensen-Dalsgaard, J., et al. 1996, *Science*, 272, 1286
- Dziembowski, W. A., & Goode, P. R. 2004, *ApJ*, 600, 464
- Dziembowski, W. A., & Goode, P. R. 2005, *ApJ*, 625, 548
- Kuhn, J. R., Bush, R. I., Emilio, M., & Scherrer, P. H. 2004, *ApJ*, 613, 1241
- Laclare, F., Delmas, C., Coin, J.-P., & Irbah, A. 1996, *Sol. Phys.*, 166, 211

- Lefebvre, S., & Kosovichev, A. G. 2005, *ApJ*, 633, L149 (LK05)
- Lefebvre, S., Kosovichev, A. G., Nghiem, P., Turck-Chièze, S., & Rozelot, J. P. 2006, in *Proc. of SOHO 18/GONG 2006/HelAs I meeting*, ed. K. Fletcher (ESA SP-624) (Noordwijk: ESA), in press
- Noël, F. 2004, *A&A*, 413, 725
- Reis Neto, E., Andrei, A. H., Penna, J. L., Jilinski, E. G., & Puliaev, S. P. 2003, *Sol. Phys.*, 212(1), 7
- Schou, J., Kosovichev, A. G., Goode, P. R., & Dziembowski, W. A. 1997, *ApJ*, 489, L197
- Sofia, S., Heaps, W., & Twigg, L. W. 1994, *ApJ*, 427, 1048
- Sofia, S., Basu S., Demarque, P., Li, L., & Thuillier, G. 2005, *ApJ*, 632, L147 (SBDLT05)
- Tikhonov, A. N., & Arsenin, V. Y. 1977, *Solutions of ill-posed problems*, Washington D. C.: Winston
- Winnick, R. A., Demarque, P., Basu, S., & Guenther, D. B. 2002, *ApJ*, 576, 1075
- Wittmann, A. D., & Bianda, M. 2000, in *Proc. of the 1st Solar and Space Weather Euro-conference*, ed. A. Wilson (ESA SP-463) (Noordwijk: ESA), 113

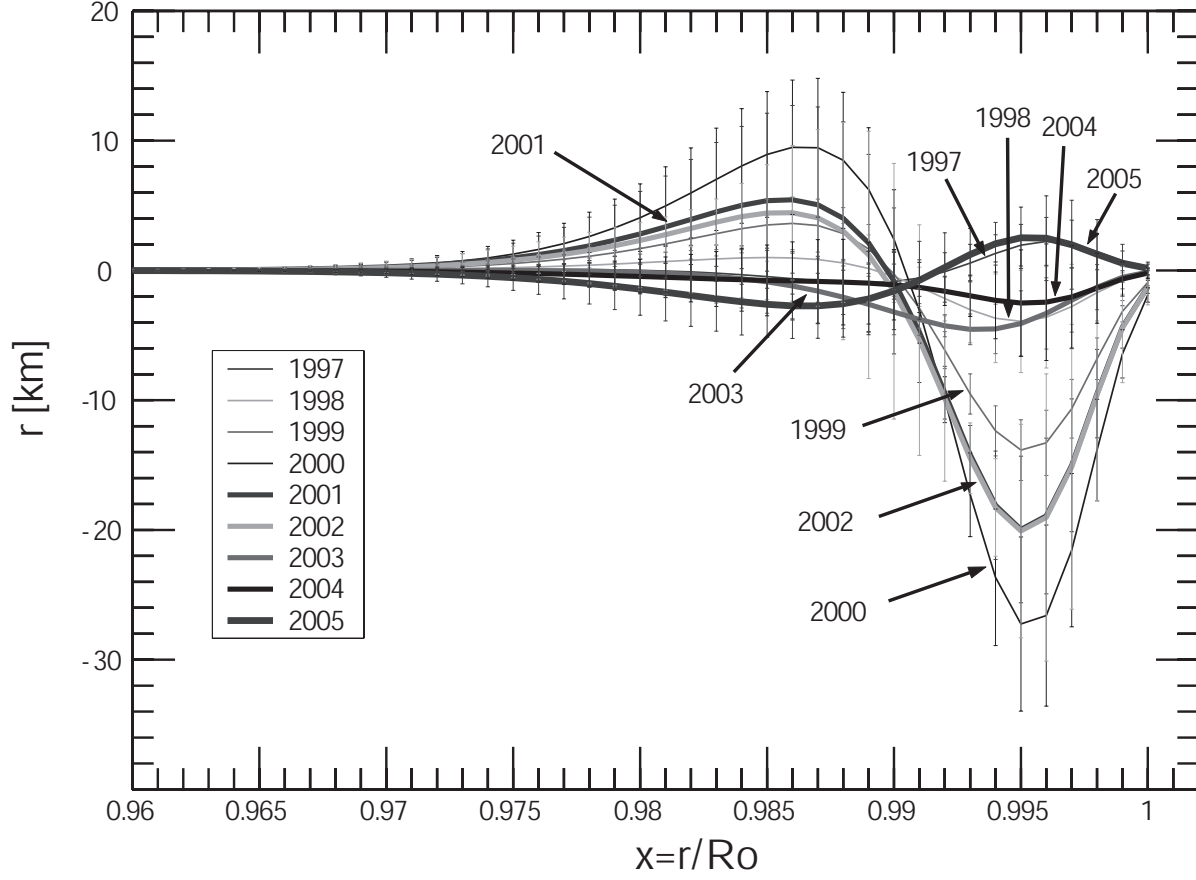


Fig. 1.— Radial variation $\langle \delta r \rangle$ as a function of the fractional radius $x = r/R_\odot$, obtained as a solution of the inversion of f -mode frequencies by a least-squares regularization technique. The reference year is 1996. The error bars are the standard deviation after average over a set of random noise added to the relative frequencies. The averaging kernels for this inversion are well localized between 0.985 and 0.996, with a typical half-width of about 0.003.

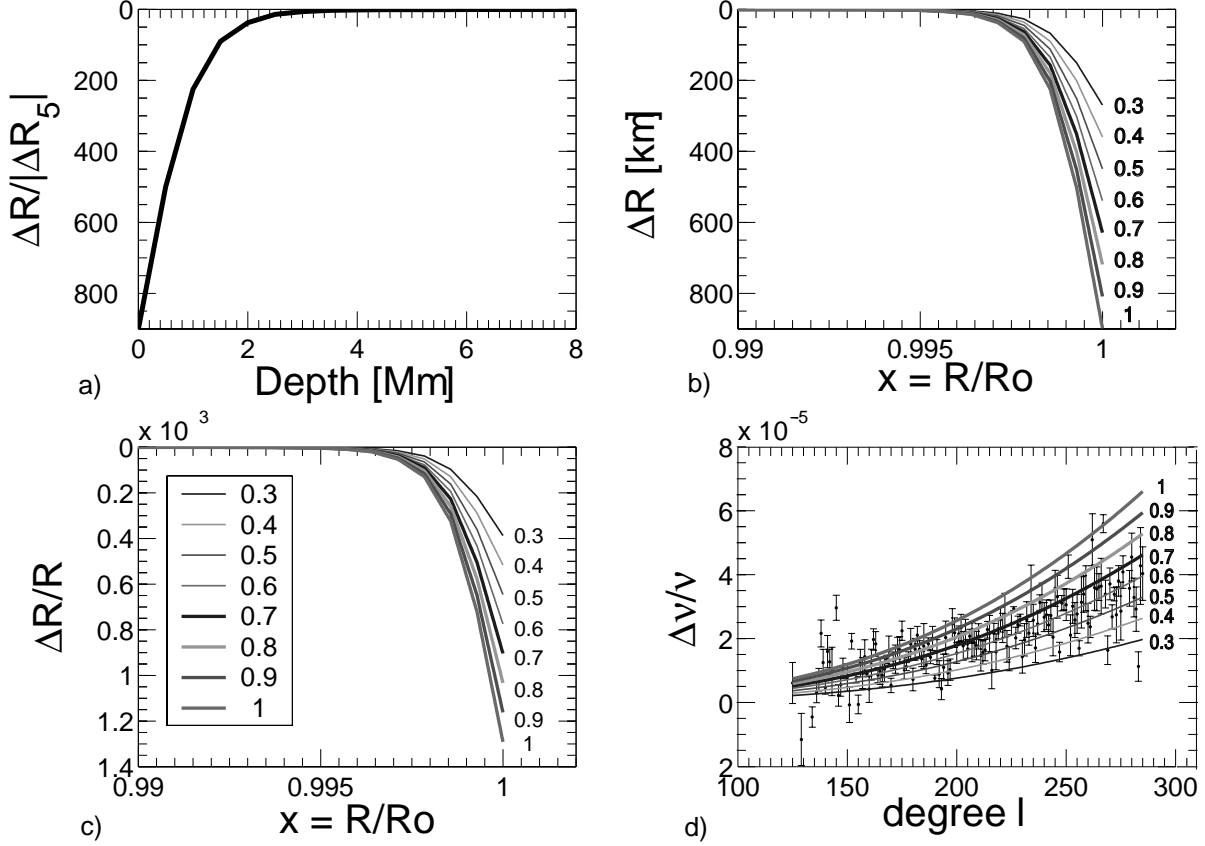


Fig. 2.— *a)*: The ratio between radius change as a function of depth below the photosphere and radius change at 5 Mm as a function of depth in Mm, i.e. same figure as in top panel of Figure 3 of Sofia et al. (2005). *b)*: Computed ΔR with different $|\Delta R_{5Mm}|$ (see the legend expressed in km in panel *c*) as a function of the fractional radius $x = R/R_\odot$. *c)*: Computed $\Delta R/R$ with different ΔR_{5Mm} from legend (in km) as a function of the fractional radius x . *d)*: Observed $\Delta \nu/\nu$ in black point with errorbars and computed integrated $\Delta \nu/\nu$ as a function of the degree l , using Eq. 1 and $\Delta R/R$ from left bottom panel.